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Fundamental steps of the group velocity for slow surface polaritons in the two-dimensional electron gas in a high magnetic field

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Abstract. A new type of collective electromagnetic excitation, namely surface polaritons (SP)—in a 2D electronic layer in a high magnetic field, is predicted. We have found the spectrum, damping, and polarization of the SP over a wide range of frequencies ω and wavevectors $\boldsymbol{k}$. It is shown that near the cyclotron resonance (CR) ($\omega \sim \Omega = eB/mc$) the phase velocity of the SP is drastically slowed down and the group velocity is quantized into fundamental steps defined by the fine-structure constant $\alpha = e^2/\hbar c$. In the vicinity of a CR subharmonic ($\omega \sim 2\Omega$), negative (anomalous) dispersion of the SP occurs. The relaxation of electrons in the 2D layer gives rise to a new dissipative collective threshold-type mode of the SP. We suggest a method for calculating the kinetic coefficients for the 2D electronic layer in a high magnetic field, using the Wigner distribution function formalism, and determine their spatial and frequency dispersions. Using this method we have calculated the lineshapes of the CR, which are in good agreement with experimental data.

1. Introduction

Since the discovery of the quantum Hall effect (QHE) [1–3], a number of authors have investigated the weak damping of collective electromagnetic waves in 2D electronic layers in a strong magnetic field $\boldsymbol{B}$ [3, 4, 5, 6]. The quantization of the Hall conductivity and the vanishingly small dissipative (longitudinal) conductivity lead, under the QHE conditions, to spatial and frequency dispersion in the system and hence to the generation of an unusually slow collective wave, whose dispersion characteristics are also quantized.

In this paper a new type of collective electromagnetic excitation in a 2D electronic system (2DES) in a high magnetic field (under the QHE conditions [9]) is predicted, namely, the slow surface polaritons (SP).

Surface polaritons are electromagnetic waves that propagate along a flat surface separating two dielectric media and whose amplitudes decay exponentially with increasing distance from the surface into either medium. In recent years considerable interest arose in the study of the SP in 2DES [7], and superlattices (see, e.g., [8]). In [5] the magnetoplasma oscillation in 2DES in a magnetic field was investigated. In that it was shown that near the cyclotron resonance (CR) the SP slowed down, but neither that work [5] nor other studies considered the effects determined by the Hall conductivity quantization in a high magnetic field and also neither did they consider the conductivity spatial dispersion effects.

In our paper we calculate the spectrum, damping, and polarization of that wave over a wide range of frequencies ω and wavevectors $\boldsymbol{k}$. The phase velocity of the SP is drastically slowed down near the principal cyclotron resonance ($\omega \sim \Omega$, where $\Omega = eB/mc$ is the cyclotron frequency) and their group velocity is quantized into fundamental jumps, whose magnitude is determined by the fine-structure constant $\alpha = e^2/\hbar c$. The number of slow SP modes is determined by the magnitude of the Landau-level filling factor $\aleph = \pi\ell^2 n$ (where $\ell = (c\hbar/eB)^{1/2}$ is the magnetic length, and n the density of 2D electrons), i.e., by the value of the quantized Hall conductivity. In the vicinity of the CR subharmonic ($\omega \sim 2\Omega$), the negative (anomalous) dispersion of SP (see figure 3, later) occurs. Besides this, a new type of SP appears near the CR, which is dissipative in nature. The condition of existence of that additional SP is determined by the quantized threshold criterion, which allows determination of the relaxation frequency at low temperature to an accuracy of α (see equation (3.17)).

The QHE has been intensively studied with the use of methods of modern condensed-matter theory [3]. The integer QHE (IQHE) is thought to be caused by localization of electrons in the two-dimensional systems, and the fractional QHE (FQHE) is due to electron–electron interaction, which leads to the generation of a correlated many-particle ground state [3] at distinct fractional values of the Landau-level filling factor $\aleph$. As this takes place, the paradox lies in the fact that the presence of 'dirt' is the necessary condition for the localization–delocalization phase transition effect—although it is well known that for the observation of QHE, and, particularly FQHE, the use of perfect samples with a high mobility is required. QHE is one of the problems of the *postmodern quantum mechanics* (to use the term of Harris), discussed in four papers in *Physics Today* in 1993 [10]. The technological advances of the last decade have led to fabrication being achieved of two-dimensional systems in which the ballistic (or quasiballistic) transport with large mean free paths can be realized.

From the above reasoning we suggest a simple method, which uses almost solely the Pauli principle, for the description of the kinetics of electrons in 2D systems placed in a strong quantizing magnetic field $\boldsymbol{B}$. By using the Wigner distribution function [11] we derive the kinetic characteristics of a 2D electronic gas under the QHE conditions and find their spatial and frequency dispersions. By means of these results we can adequately describe the d.c. effects of IQHE, as well as the lineshape of the CR under the IQHE conditions and hence the dynamics of collective electromagnetic excitations under QHE conditions.

The article is organized as follows. In section 2 we use the Wigner distribution function for describing the transport phenomena in a 2DES under QHE conditions. We calculate the conductivity tensor with a spatial and frequency dispersion. In section 3 we present the electrodynamics in 2DES in the high magnetic fields (the QHE effect). We derive the dispersion relation for electromagnetic surface waves and discuss the dispersion, polarization and damping for the quantized SP in this system. We conclude the paper with a brief summary of results and possible applications (section 4).

2. Transport in the 2DES in a high magnetic field

To find the conductivity tensor accounting for the spatial and frequency dispersion in a 2D electronic gas placed in a high quantizing magnetic field $\boldsymbol{B}$ (under QHE conditions) oriented normally to the 2D layer (see figure 1), we will apply the Wigner distribution function [11, 12, 13]:

$$f_{\boldsymbol{p}}^{W}(\boldsymbol{r}) = \int \mathrm{d}\boldsymbol{r}'\ \mathrm{Tr}\{\hat{\rho}\ \exp\left[-\mathrm{i}\left(\boldsymbol{p} + \frac{e}{c}\boldsymbol{A}(\boldsymbol{r})\right)\boldsymbol{r}'\right]\psi^{+}(\boldsymbol{r} - \boldsymbol{r}'/2)\psi(\boldsymbol{r} + \boldsymbol{r}'/2)\}. \tag{2.1}$$

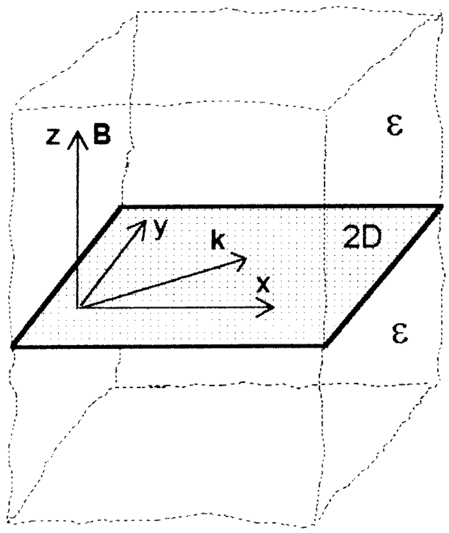

Figure 1. The geometry of the structure of the 2D electronic layer embedded in a dielectric medium with dielectric constant ε.

Here $\hat{\rho}$ is the statistical operator of the system; $\psi^{+}(\boldsymbol{r})$ and $\psi(\boldsymbol{r})$ are the Fermi operators for generation and annihilation, respectively, of particles at point $\boldsymbol{r}$; and $\boldsymbol{A}$ is the vector potential of the electromagnetic field. In the case where the scale of the spatial inhomogeneity exceeds both the radius of interaction between the particles and the de Broglie electron wavelength, the kinetic equation for the Wigner distribution function, equation (2.1), takes a form [11–13] equivalent to the classical kinetic equation:

$$\frac{\partial f_{\boldsymbol{p}}^{W}}{\partial t} + \boldsymbol{v}\frac{\partial f_{\boldsymbol{p}}^{W}}{\partial \boldsymbol{r}} + e\left\{\boldsymbol{E} + \frac{1}{c}[\boldsymbol{v}, \boldsymbol{B}]\right\}\frac{\partial f_{\boldsymbol{p}}^{W}}{\partial \boldsymbol{p}} = \hat{I}\{f_{\boldsymbol{p}}^{W}\}. \tag{2.2}$$

Here $\boldsymbol{E}$ and $\boldsymbol{B}$ are the electric field and the magnetic induction vectors; e is the charge on the electron, and $\boldsymbol{v}$ is the velocity of the conduction electrons.

In the case under consideration, if the 2D electronic system is infinite in the xy-plane (see figure 1), then the typical scale of inhomogeneity is the wavelength k^{-1} of the collective electromagnetic wave. Thus, the existence criteria for equation (2.2) are $k \ll n^{1/2}$ (since with weak screening $n^{-1/2}$ is the characteristic length of interaction between the particles) and $k\ell \ll 1$ (since in a strong magnetic field the magnetic length $\ell = (c\hbar/eB)^{1/2}$ represents the de Broglie wavelength of electrons). The collision integral, $\hat{I}\{f_{\boldsymbol{p}}^{W}\}$, differs essentially from the classical collision integral, since the quantum transitions accounted for by $\hat{I}\{f_{\boldsymbol{p}}^{W}\}$ reflect the character of the statistics obeyed by the particles, and the distinction of the Wigner distribution function from the classical one [12]. The equilibrium Wigner distribution function sets the collision integral, $\hat{I}\{f_{\boldsymbol{p}}^{W}\}$, to zero. The equilibrium Wigner function can be expressed via its value for an equilibrium ensemble of quantum states of an electron in a magnetic field $\boldsymbol{B}$. By using the definition given as equation (2.1) and substituting the wavefunctions of an electron in an electromagnetic field into equation (2.1), we obtain for the spinless electrons [12, 13]

$$
\begin{aligned}
f_0(\epsilon) &= \sum_{s=o}^{\infty} n_F\left[\frac{\hbar\Omega(s+\frac{1}{2})-\mu}{T}\right]\Gamma_s\left(\frac{\epsilon}{\hbar\Omega}\right) \\
\Gamma_s(x) &= 2(-1)^s \exp(-2x) L_s^{(0)}(4x) \\
n_F(x) &= (1+\mathrm{e}^x)^{-1}.
\end{aligned}
\tag{2.3}
$$

Here $\epsilon = p^2/2m$ is the energy of 2D electrons and $L_s^{(0)}(x)$ the Laguerre polynomial. If we replace the summation over s by an integration, then for $\hbar\Omega \ll T$ (T is the temperature) equation (2.3) transforms into an equilibrium Fermi distribution function (μ is the chemical potential).

With the knowledge of the equilibrium Wigner distribution function we can describe all of the thermodynamical relations. In this paper we will consider the 2DES when the chemical potential μ is constant over the entire system. The relation between the electron density n and the chemical potential μ in a strong magnetic field can be found, as usual, from the normalization condition. We are considering the 2DES with a fixed value of chemical potential μ. The real 2DES is in fact extremely inhomogeneous in one of the directions of the 3D system. Examples of two-dimensional electron systems occur in the inversion layers of metal–oxide–semiconductor field-effect transistors (MOSFETs) and $\mathrm{GaAs}-\mathrm{Al}_x\mathrm{Ga}_{1-x}\mathrm{As}$ heterostructures at low temperature. In such systems the chemical potential μ is determined for the 2DES as an equilibrium value of μ for the extremely inhomogeneous three-dimensional systems. The density n of 2D electrons is the average of $f_0(\epsilon)$ (equation (2.3)). As a result we obtain after the averaging

$$
\aleph = \pi\ell^2 n = \sum_{s=0}^{\infty} n_F\left[\frac{\hbar\Omega(s+\frac{1}{2})-\mu}{T}\right]. \tag{2.4}
$$

It is shown that the Landau-level filling factor $\aleph$ assumes only positive integer values if $T \ll \hbar\Omega, \mu$. The fact that the filling factor $\aleph$ can assume only integer values leads to the IQHE. However, equation (2.4) contains a contradiction. Indeed, why should the ratio of two independent values—which the electron density n and the magnetic induction $\boldsymbol{B}$ in the sample are—take only integer values? It is well known that the mean value of a microscopic magnetic field is the magnetic induction $\boldsymbol{B}$. The value of $\boldsymbol{B}$ in the sample should be found from the formula $\boldsymbol{H} = \boldsymbol{B} - 4\pi\boldsymbol{M}(\boldsymbol{B})$, where $\boldsymbol{H}$ is the external magnetic field and $\boldsymbol{M}(\boldsymbol{B})$ the magnetic moment. If the temperature is not too low, then $M(\boldsymbol{B}) \ll B$ and the magnetic induction value B differs only slightly from H. As temperature is decreased, the amplitude of oscillations of the magnetic moment $\boldsymbol{M}(\boldsymbol{B})$ increases and a situation appears in which regions of H-values corresponding to three different values of B show up. This ambiguity indicates an instability of states similar to that found on the Van der Waals curve of the equation of state [14–16]. In other words, at such values of the external magnetic fields, diamagnetic phase transitions take place in the system, at which an inhomogeneous state (domain type and (or) periodic structures) appears in the system, when the magnetic induction B and the electron density n become coordinate-dependent functions. In the vicinity of such phase transitions and for the inhomogeneous states, the scaling-type dependencies of the conductivity on the magnetic field and singularities at the fractional values of the Landau-level filling factor $\aleph$ should appear due to the scaling-type invariance (see [14, 15, 16]). This kind of behaviour should result in singularities in the Hall conductivity and in a longitudinal (dissipative) conductivity of the 2D electronic gas at the fractional values of the filling factor $\aleph$, i.e., lead to the FQHE. In particular, when the magnetic induction and the electron density are characterized by an inhomogeneous structure, the splitting of states and the additional gaps might originate in the electron spectrum, and the spectrum degeneracy in the orbit centre coordinate in the magnetic field may be lifted.

Besides this, an additional drift of electrons [17] can arise in a weakly inhomogeneous magnetic field which can give an additional contribution to the Hall current. However, far away from the diamagnetic phase transition [14, 15, 16] equilibrium states exist; the electron density is independent of the coordinates, and the magnetic induction $\boldsymbol{B}$ assumes a sufficient value to satisfy relation (2.4), and the Landau-level filling factor is an integer value, i.e., the IQHE conditions are met. Thus, in this paper we will analyse the electron kinetics under the IQHE conditions.

The formalism used in this paper is based on the assumption that the kinetic equation for the Wigner distribution function can also involve the collision integral. It is well known (see, e.g., [7]) that this can be done if the crystal periodicity violation, which is the source of electron scattering, does not (or does weakly) distort the electron spectrum of the ideal crystal. This way a weak disorder can be taken into account within the WDF formalism. Of course, the impurity scattering in collision integral form for the Wigner distribution function must be treated self-consistently using (e.g.) the self-consistent Born approximation, which is the simplest treatment that is free from divergences. In other words, the collision integral can be described in terms of a relaxation frequency depending on electron energy. In particular, the formalism allows one to automatically take into account the effect of Landau-level broadening due to collisions with impurities. (In the case where the impurity potential is a short-range one, the collision frequencies are not energy dependent.) It is clear that for the case of high mobility current carriers and sufficiently high frequency of the electromagnetic field, the approximation for the collision integral is justified. The forms of the electron–phonon and electron–impurity collision integrals, equation (2.3), are too complicated [12]. However, we will consider here the effects determined by the linear response to the electric field. In this case the distribution function, $f_{\boldsymbol{p}}^W$, can be found in an approximation linear in the external field, $\boldsymbol{E}$. It is well known [12] that for a sample with a high electron mobility and for describing high-frequency effects ($\omega \gg \nu$), the collision integral can be represented in the τ-approximation, where the mean free path time τ is determined by the momentum relaxation frequency, being a function of the electron energy, ϵ. In other words, for this quasiballistic regime we can find the Wigner distribution function in the form

$$f_{\boldsymbol{p}}^W = f_0(\epsilon) + f_1 \tag{2.5}$$

where $f_0(\epsilon)$ is the equilibrium distribution function in a high magnetic field, equation (2.3), and $f_1(t, \boldsymbol{p}, \boldsymbol{r})$ is the correction to the Wigner distribution function, which is determined by the electric field $\boldsymbol{E}$. The collision integral, $\hat{I}\{f_{\boldsymbol{p}}^W\}$, will be written as

$$\hat{I}\{f_{\boldsymbol{p}}^W\} = -\nu(\epsilon) f_1. \tag{2.6}$$

In the general case we will assume the electric field to be a function of coordinates and time. Then for the Fourier transform of the current density, $\boldsymbol{j}$, and the electric field, $\boldsymbol{E}$, one obtains the linear relations

$$j_\alpha(\omega, \boldsymbol{k}) = \sigma_{\alpha\beta}(\omega, \boldsymbol{k})\, \mathcal{E}_\beta(\omega, \boldsymbol{k}) \tag{2.7}$$

$$E_\alpha(\boldsymbol{r}, t) = \int \frac{\mathrm{d}^2k\ \mathrm{d}\omega}{(2\pi)^3}\ \mathcal{E}_\alpha(\omega, \boldsymbol{k})\, \mathrm{e}^{\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)} \tag{2.8}$$

$$\mathcal{E}_\alpha(\omega, \boldsymbol{k}) = \int \mathrm{d}^2k\ \mathrm{d}\omega\ E_\alpha(\omega, \boldsymbol{k})\, \mathrm{e}^{\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}. \tag{2.9}$$

The Fourier transform of the conductivity tensor, $\sigma_{\alpha\beta}(\omega, \boldsymbol{k})$, accounting for the spatial and frequency dispersion, can be found by using the Wigner distribution function, equations (2.2), (2.3) and (2.5). Thus, in the general case we have

$$\sigma_{\alpha\beta}(\omega, \boldsymbol{k}) = \frac{2e^2}{h} \sum_{s=0}^{\infty} n_F \left\{ \frac{\hbar\Omega(s+\frac{1}{2}) - \mu}{T} \right\} \int_0^{\infty} \mathrm{d}\xi \, \frac{(\pi^2\xi/2)}{\sinh[\pi\gamma(\xi)]} \frac{\partial \Gamma_s(\xi)}{\partial \xi} D_{\alpha\beta}(\xi). \tag{2.10}$$

Here the tensor components, $D_{\alpha\beta}(\xi)$, are given by

$$\begin{aligned} D_{xx}(\xi) &= m_1(\xi) + m_2(\xi) + 2m_3(\xi)\cos 2\beta \\ D_{yy}(\xi) &= m_1(\xi) + m_2(\xi) - 2m_3(\xi)\cos 2\beta \\ D_{xy}(\xi) &= -\mathrm{i}m_1(\xi) + \mathrm{i}m_2(\xi) + 2m_3(\xi)\sin 2\beta \\ D_{yx}(\xi) &= \mathrm{i}m_1(\xi) - \mathrm{i}m_2(\xi) + 2m_3(\xi)\sin 2\beta. \end{aligned} \tag{2.11}$$

The functions $m_1(\xi)$, $m_2(\xi)$ and $m_3(\xi)$ are defined by the expressions

$$\begin{aligned} m_1(\xi) &= J_{-\mathrm{i}\gamma-1}(z)J_{\mathrm{i}\gamma+1}(z) \\ m_2(\xi) &= J_{-\mathrm{i}\gamma+1}(z)J_{\mathrm{i}\gamma-1}(z) \\ m_3(\xi) &= J_{\mathrm{i}\gamma+1}(z)J_{-\mathrm{i}\gamma+1}(z) \end{aligned} \tag{2.12}$$

where

$$z = \sqrt{2\xi}kl \qquad \gamma = \frac{\nu(\xi\hbar\Omega) - \mathrm{i}\omega}{\Omega} \qquad k = \sqrt{k_x^2 + k_y^2}$$

$$\cos\beta = \frac{k_x}{k} \qquad \sin\beta = -\frac{k_y}{k}.$$

Note that the Fourier transform of the conductivity, equations (2.10)–(2.12), contains terms proportional to $\cos 2\beta$ and $\sin 2\beta$, whose appearance results from the two dimensionality of the electronic gas. They violate the symmetry of the kinetic coefficients for the Fourier transform of $\sigma_{\alpha\beta}(\omega, \boldsymbol{k})$. Such polarizable terms may be important for the polarization of electromagnetic eigenoscillations in 2D electronic systems. The integration over $\mathrm{d}^2\boldsymbol{r}$ re-establishes the symmetry.

One can easily see that generally the conductivity tensor experiences resonance oscillations, namely, of the cyclotron resonance type in the case of a strong spatial dispersion (when $k\ell^* \gg 1$, where ℓ^* is the mean free path) on the multiple harmonics (subharmonics of the CR when $\omega = s\Omega$, $s = 1, 2, \ldots$). However, the case of a strong spatial dispersion can be practically realized only at very high frequencies. As is easy to see from equation (2.3) and from the expression for the conductivity tensor, the strong quantization in a magnetic field 'destroys the Fermi surface' and the conductivity is due to all of the electrons with various energies. Thus, the frequency dispersion of the conductivity (and the spatial dispersion as well) is in essence defined by the form of the function $\nu = \nu(\epsilon)$. At low frequencies, when $\omega \leqslant \nu(\epsilon)$, it might be an 'indicator' of the function $\nu(\epsilon)$. It is also clear that the lineshape of the CR can be essentially dependent on the function $\nu = \nu(\epsilon)$ [15, 18]. The form of the longitudinal d.c. conductivity ($\sigma_{xx} = \sigma_{yy}$, when $\omega = 0$, $k = 0$) is also drastically dependent on the type of the function $\nu = \nu(\epsilon)$ [18]. The lineshape of the CR can be changed if the electron effective mass $m = m(\epsilon)$ depends on the electron energy ϵ, when the dispersion law of conduction electrons differs from the quadratic one [19]. In this paper we will assume that the mobility is very high and we will find the conductivity when the relaxation frequency is an effective constant, i.e., $\nu =$ constant [20]. We will consider the most realistic case, where the spatial dispersion is sufficiently weak, i.e.,

$$k\ell \ll 1. \tag{2.13}$$

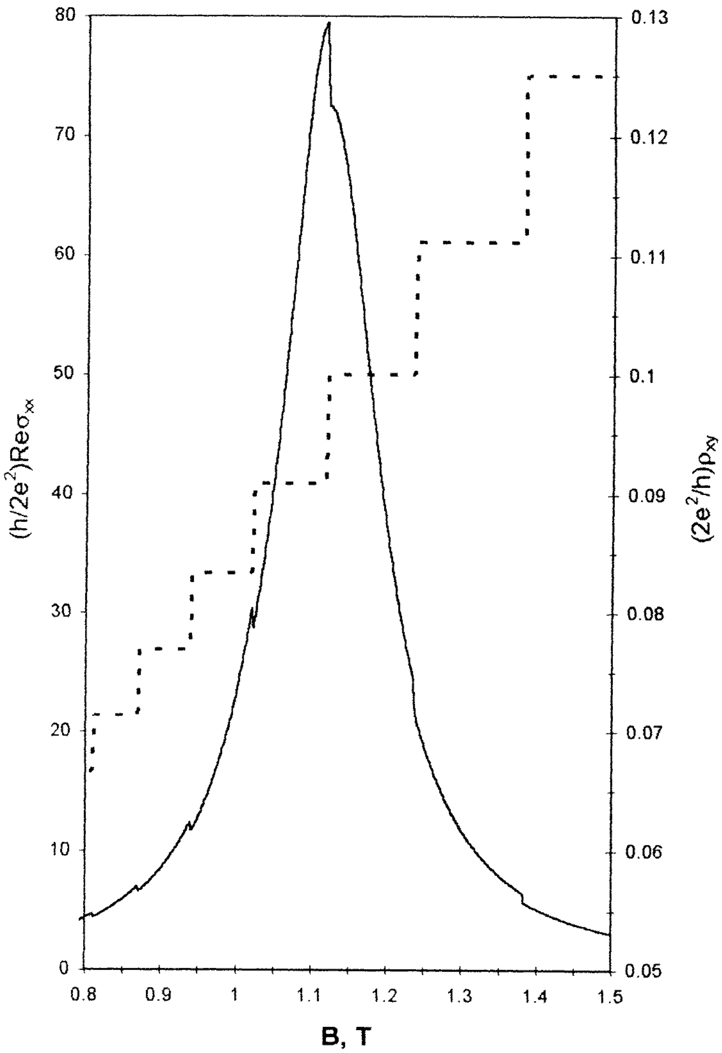

Figure 2. The lineshape of the cyclotron resonance in a 2DES under the QHE conditions.

Then the Fourier transform of the conductivity tensor can be obtained in the form

$$\sigma_{\alpha\beta} = \frac{2e^2}{h}\frac{\aleph}{1+\gamma^2}\left\{B_{\alpha\beta} - \frac{(k\ell)^2}{2\gamma}\left(1+\frac{\aleph}{2}\right)C_{\alpha\beta}\right\} \tag{2.14}$$

where

$$\begin{aligned}
&B_{xx} = B_{yy} = \gamma && B_{xy} = -B_{yx} = 1 \\
&C_{xx} = a + \cos 2\beta && C_{yy} = a - \cos 2\beta \\
&C_{xy} = -b - \sin 2\beta && C_{yx} = b - \sin 2\beta \\
&a = \frac{2(\gamma^2+2)}{\gamma^2+4} && b = \frac{6\gamma}{\gamma^2+4}.
\end{aligned} \tag{2.15}$$

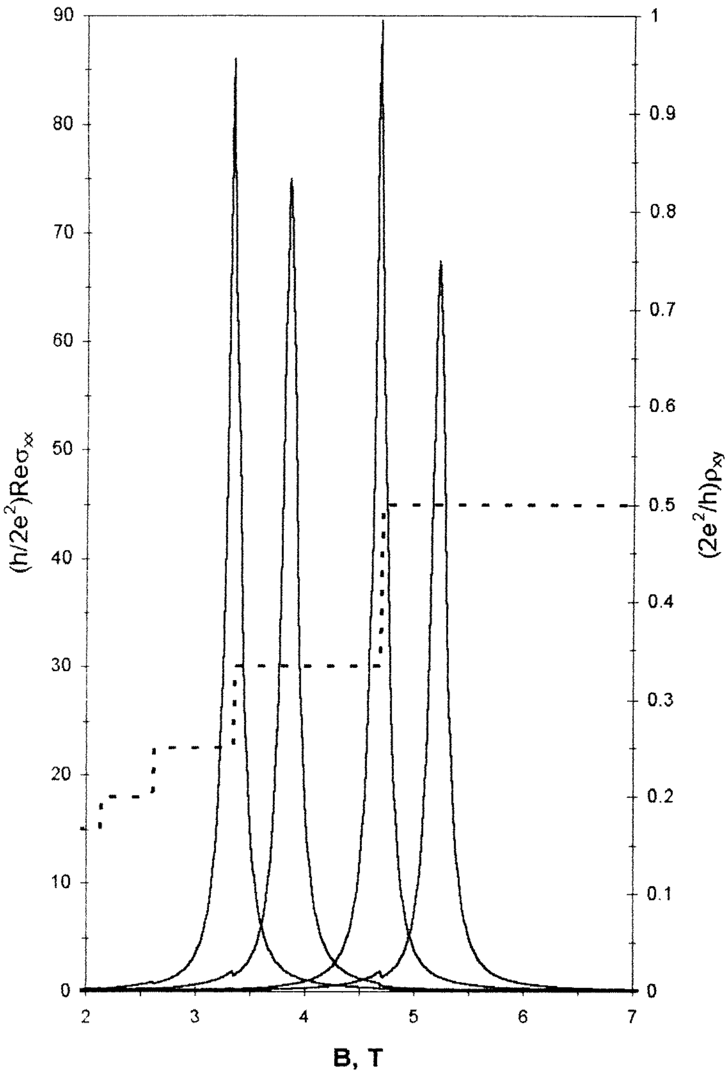

Figure 2. (Continued)

Let us summarize the formulae for the resistance tensor in the d.c. case, when $\omega \to 0$, $k \to 0$ (the IQHE):

$$\rho_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2} = \frac{h}{2e^2}\frac{\gamma}{\aleph} \tag{2.16}$$

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2} = \frac{h}{2e^2}\frac{1}{\aleph}. \tag{2.17}$$

As can be seen, equations (2.16) and (2.17) are good enough to describe the classical picture of the IQHE [3, 21], even for $\nu =$ constant. Equation (2.14) and equation (2.15) show that if $\nu =$ constant then the relation $\rho_{xx}/\rho_{xy} = \gamma = \nu/\Omega$ should hold—which can be observed under the IQHE conditions. The deviation from this simple relation demonstrates the energy dependence of the relaxation frequency, $\nu = \nu(\epsilon)$ [21]. Here we will show the lineshape of the CR (the high-frequency absorption $\sim\mathrm{Re}\,\sigma_{xx}$) for various frequencies, ω, as a function of the magnetic field (see figure 2). Obviously, the lineshape of the CR is highly sensitive to the CR position. In the case when the line centre is located at the centre of the IQHE plateau, it shows kinks at the points where the jumps of the QHE occur. Figure 2 (left) shows the lineshape of the cyclotron resonance in the 2DES calculated for GaAs/GaAlAs from formula (2.14), for the temperature $T = 50$ mK ($k \to 0$) and $\nu = 2 \times 10^{11}$ s^{-1}. The wide resonance line, which captures several steps of the QHE for the rather low frequency ω, when the CR occurs at a quite small value of the magnetic field. It is shown that the additional structure of the line came about from the interference of singularities of the CR and the QHE. If the centre of the CR line is located near a QHE step, then the amplitude of the CR is increased. Figure 2 (right) shows the narrow resonance lines, which are found for the higher frequencies ω located in the range of one step of the QHE. It is shown that the structure of the line whose centre is in the centre of the Hall plateau exists on the wings of a line at the values of the magnetic field $\boldsymbol{B}$ corresponding to the jump of the Landau-level filling factor $\aleph$. The amplitude of the CR line is increased when the centre of the CR line is located at the point of the jump of the Landau-level filling factor $\aleph$ (the lines for the frequencies $\omega_3 = 1.00 \times 10^{13}$ s^{-1}, $\omega_5 = 1.35 \times 10^{13}$ s^{-1}). Such features of the CR line were observed by a number of authors [22].

3. Electrodynamics of 2DES in a high magnetic field

The propagation of electromagnetic waves through systems with a 2D electronic gas in the dielectric environment, placed in a strong magnetic field (see figure 1), is described by the Maxwell equations for the scalar and vector potentials in the Lorentz gauge, namely

$$\boldsymbol{\nabla}\cdot\boldsymbol{A} + \frac{\epsilon}{c}\frac{\partial\varphi}{\partial t} = 0. \tag{3.1}$$

The potentials satisfy the usual wave equation [23, 24]

$$\left[\nabla^2 - \frac{\epsilon}{c^2}\left(\frac{\partial}{\partial t}\right)^2\right]\varphi(\boldsymbol{r}, t) = -\frac{4\pi}{\epsilon}\rho_{tot}(\boldsymbol{r}, t) \tag{3.2}$$

$$\left[\nabla^2 - \frac{\epsilon}{c^2}\left(\frac{\partial}{\partial t}\right)^2\right]\boldsymbol{A}(\boldsymbol{r}, t) = -\frac{4\pi}{c}\boldsymbol{j}_{tot}(\boldsymbol{r}, t). \tag{3.3}$$

These values are related to the fields in equation (2.2) by

$$\boldsymbol{E} = -\boldsymbol{\nabla}\varphi - c^{-1}\frac{\partial\boldsymbol{A}}{\partial t} \qquad \boldsymbol{B} = \mathrm{rot}\,\boldsymbol{A}. \tag{3.4}$$

Here $\rho_{tot} = \rho_{ex} + \rho$ is the total charge density in the system and $\boldsymbol{j}_{tot} = \boldsymbol{j}_{ex} + \boldsymbol{j}$ the total current density; ρ_{ex} and $\boldsymbol{j}_{ex}$ are the external charge and current densities, respectively. In the system under consideration $\rho \sim\ j \sim \delta(z)$ [24] , so the charges and currents exist only in the 2D electronic layer, and external currents and charges are absent from the system: $\rho_{ex} = j_{ex} = 0$. In this case the potentials $\boldsymbol{A}$ and φ can be found from the homogeneous set of equations (3.2) and (3.3). Using the Fourier transformation, equation (2.7), and taking into account $\boldsymbol{j}(\omega, \boldsymbol{k}, z) = \boldsymbol{j}(\omega, \boldsymbol{k}, 0)\delta(z)$, we obtain

$$\boldsymbol{A}(\omega, \boldsymbol{k}, z) = \boldsymbol{A}_0(\omega, \boldsymbol{k})\mathrm{e}^{-p|z|} \tag{3.5}$$

where

$$p = \sqrt{k^2 - \frac{\omega^2}{c^2}\varepsilon}$$

and Re $p > 0$. In other words, the system supports eigenoscillations in the form of a surface wave pressed up against the 2D electronic layer (see figure 1). In this case the component $A_z = 0$, while the scalar potential can be found from the Lorentz gauge (2.17):

$$\boldsymbol{k} \cdot \boldsymbol{A} = \frac{\omega\varepsilon}{c}\varphi.$$

The current density in the 2D electronic layer can be represented in the form

$$\boldsymbol{j}_\alpha(\omega, \boldsymbol{k}) = \mathrm{i}\frac{\omega}{c}\sigma_{\alpha\beta}(\omega, \boldsymbol{k})\left[A_\beta(\omega, \boldsymbol{k}, 0) - \frac{c^2}{\varepsilon\omega^2}k_\beta k_\gamma A_\gamma(\omega, \boldsymbol{k}, 0)\right]. \tag{3.6}$$

Thus, the dispersion relation is found from the condition

$$\boldsymbol{A}(\omega, \boldsymbol{k}, 0) = \frac{2\pi}{cp}\,\boldsymbol{j}(\omega, \boldsymbol{k}, 0). \tag{3.7}$$

This dispersion relation takes the form

$$\boldsymbol{D} = \mathrm{Det}\left\{\delta_{\alpha\beta} - \frac{2\pi\mathrm{i}\omega}{c^2 p}\left[\sigma_{\alpha\beta}(\omega, \boldsymbol{k}) - \frac{c^2}{\epsilon\omega^2}\sigma_{\alpha\gamma}(\omega, \boldsymbol{k})k_\gamma k_\beta\right]\right\} = 0 \tag{3.8}$$

where $\delta_{\alpha b}$ is the Kronecker delta.

Figure 3 presents the dispersion curves of the SP propagating in the system of figure 1. The dispersion curves for the surface polariton on the boundary of a 2DES were calculated for the various values of the Landau-level filling factor $\aleph$ ($\aleph = 1$, $\aleph = 5$, and $\aleph = 10$). The 2DES is realized by the heterostructure GaAs/GaAlAs; the effective mass $m = 0.068m_0$; and $\varepsilon = 12$. The y-axis gives the real part of the frequency, and the x-axis gives the wavenumber. It is seen that the spectrum of the SP is gapless at low frequencies ($\omega \ll \Omega$), and that they exist both in the low-frequency region $\omega < \Omega$ and in the high-frequency region $\omega > \Omega$. In the low-frequency region, far away from the CR, the phase velocity of the SP is close to the light velocity $v_d = c/\sqrt{\varepsilon}$ in the dielectric, which surrounds the 2D electronic layer. In the high-frequency region and in the vicinity of the principal CP ($\omega \sim \Omega$), the phase velocity of the SP drastically decreases and they are transformed into slow waves. In the frequency range $\Omega < \omega < 2\Omega$, where $\ell^{-1} \gg k \gg (\omega/c)\sqrt{\varepsilon}$, one can neglect the retardation effect and the spatial dispersion in the conductivity tensor of equation (2.14). The dispersion relation can be brought into the form

$$\omega^2 = \Omega^2 + \frac{2\pi v_n k\Omega}{\varepsilon} \tag{3.9}$$

where $v_n = (2e^2/h)\aleph$. It is easy to see that with $\Omega \gg v_n k/\varepsilon$ ($\aleph \sim 1$) the dispersion law of the SP is linear ($\omega \sim k$), while in the opposite case, $\Omega \gg v_n k/\varepsilon$ (or when $\aleph \gg 1$), the dispersion law is of a square-root type ($\omega \sim \sqrt{k}$).

Near the CR ($\omega \sim \Omega$), the retardation effect cannot be neglected and the dispersion law of the SP becomes

$$\omega = \Omega + \Omega\frac{v_n}{c}\left\{\frac{\pi\Omega}{cp_1}\left(\frac{p_1^2 c^2}{\epsilon\Omega^2} - 1\right) + \frac{2\pi^2 v_n}{\epsilon c}\right\} - \mathrm{i}\nu. \tag{3.10}$$

Here

$$p_1 = \sqrt{k^2 - \frac{\Omega^2}{c^2}\varepsilon}.$$

The value of the relative deceleration of the SP is determined by the fine-structure constant α. The group velocity, $\boldsymbol{v}_g = \partial\omega/\partial\boldsymbol{k}$, of the SP is quantized into fundamental steps in the vicinity of the CR:

$$\frac{v_g}{v_d} = \frac{2\sqrt{2}\alpha\aleph}{\sqrt{\varepsilon}}. \tag{3.11}$$

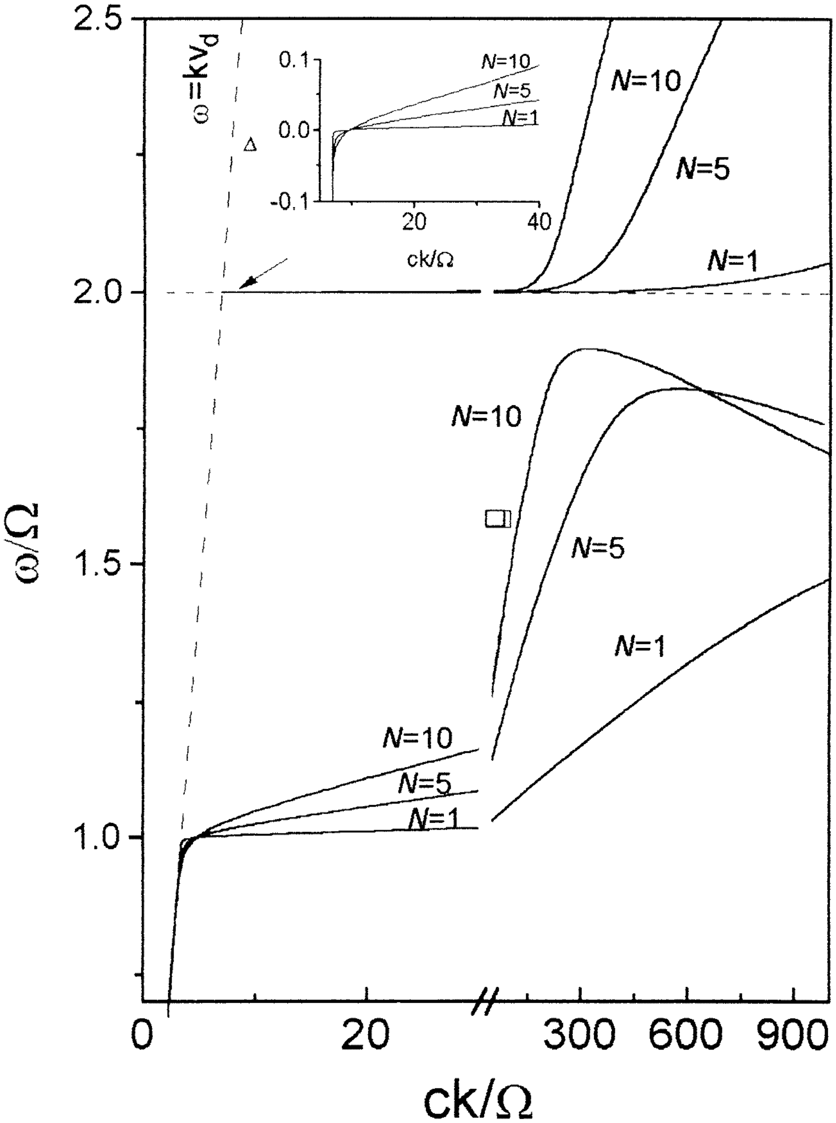

Figure 3. The dispersion curves for the surface polariton on the boundary of the 2DES calculated for various values of the Landau-level filling factor $\aleph$ ($\aleph = 1, \aleph = 5$, and $\aleph = 10$).

In other words, the deceleration of the wave near the CR is considerable, and the reason for the quantization of the group velocity is the quantization of the Hall conductivity, i.e., in fact the system possesses a fundamental parameter of the velocity dimension, the conductance quantum $2e^2/h$. At the point of the CR the character of the conductivity is changed and it becomes imaginary, i.e. reactive, and therefore the conductance becomes nondissipative, and the slow wave (the slow SP) appears. With a further increase of frequency ω, near the doubled CR ($\omega \sim 2\Omega$), the spatial dispersion effects of the conductivity equation (2.14) become noticeable, and the group velocity changes its sign and takes negative values. In this region the SP shows anomalous (negative) dispersion.

The SP spectrum near the CR subharmonic ($\omega \sim 2\Omega$) is obtained in the form

$$\omega = 2\Omega - \Omega\frac{b}{d}\left(\frac{k\ell}{2}\right)^2\left(1+\frac{\aleph}{2}\right) - \mathrm{i}\nu. \tag{3.12}$$

Here

$$b = \frac{v_n}{c}\left(\frac{4\pi}{3}\right)\left(\frac{\Omega}{cp}\right)\left(1-\frac{p_2^2c^2}{4\varepsilon\Omega}\right). \tag{3.13}$$

Also $d = 1+2b$ and

$$p_2 = \sqrt{k^2 - \frac{4\Omega^2}{c^2}\varepsilon}.$$

For $\omega > 2\Omega$ the SP propagates through the system at a velocity close to that of the SP far away from the doubled CR (see figure 3). It can be seen from equation (3.12) that the spectrum of the SP is 'strongly pressed' to the line of the CR subharmonic ($\omega = 2\Omega$). The relative dispersive width of the SP has the scale of the small parameter $\hbar\Omega/mc^2 \ll 1$ near the CR. This dispersion curve (see the inset in figure 3) of the SP starts near the fundamental mode of the light line ($\omega = kv_d$), then it is branched (the number of branches is equal to the Landau-level filling factor $\aleph$), and the group velocity is quantized in the same way as near the principal CR ($\omega = \Omega$). But the group velocity is very low, $v_g \sim v_d(\hbar\Omega/mc^2)$, in the wavenumber region where the dispersion curve is 'strongly pressed' to the line $\omega = 2\Omega$. The spectrum of the SP 'slides' near the line $k \simeq \varepsilon\Omega/v_n$. At such values of k the spectral curve is detached from the line $\omega = 2\Omega$ (see figure 3). The relative attenuation rate of the SP is of the order ν/ω and is small for samples with a high electron mobility. At low frequencies ($\omega \ll \Omega$) the SP is a wave of the TE type, changing to TM type for $\omega > \Omega$. The SP polarization is defined by the following expression:

$$\boldsymbol{A} = A_0\begin{pmatrix}1\\ q\\ 0\end{pmatrix}\mathrm{e}^{-p|z|}\,\mathrm{e}^{\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}. \tag{3.14}$$

The vector potential components are interrelated as $A_y = qA_x$, $A_z = 0$, and the scalar potential φ is given by the Lorentz gauge (3.1). The polarization parameter q is

$$q = \left\{1 - \frac{2\pi\mathrm{i}\omega}{c^2p}\left[\sigma_{xx}\left(1-\frac{c^2k_x^2}{\varepsilon\omega^2}\right) - \frac{c^2}{\omega^2}\sigma_{xy}k_xk_y\right]\right\}\Bigg/\left\{\sigma_{xy}\left(1-\frac{c^2k_y^2}{\varepsilon\omega^2}\right) - \frac{c^2}{\varepsilon\omega^2}\sigma_{xx}k_xk_y\right\}. \tag{3.15}$$

It is easy to see that the SP polarization under the QHE conditions is sensitive to the terms $\sin(2\beta)$ and $\cos(2\beta)$ of equations (2.14) and (2.15). This leads to rotation of the polarization plane as a function of the magnetic field $\boldsymbol{B}$. Far from the CR, where the spatial dispersion of conductivity is negligible, the polarization parameter q takes the simpler form

$$q = \frac{cp}{\omega}\left\{-\mathrm{i}\frac{c}{v_n}\frac{(1+\gamma^2)}{2\pi} + \frac{cp}{\varepsilon\omega}\gamma\right\}. \tag{3.16}$$

It is evident that the polarization parameter q is quantized due to the Hall quantization.

In the system under consideration (see figure 1) another SP exists, which appears near the CR ($\omega \sim \Omega$). This SP is a dissipative-type wave. The surface wave exists when $\mathrm{Re}\,p > 0$. When the relaxation frequency $\nu \neq 0$, the frequency ω is a complex value. A straightforward analysis shows that the additional (dissipative) SP mode is practically nondispersional, and the condition for its existence is determined by the threshold condition

$$\frac{\nu}{\Omega} > 2\alpha\frac{\aleph}{\sqrt{\varepsilon}}. \tag{3.17}$$

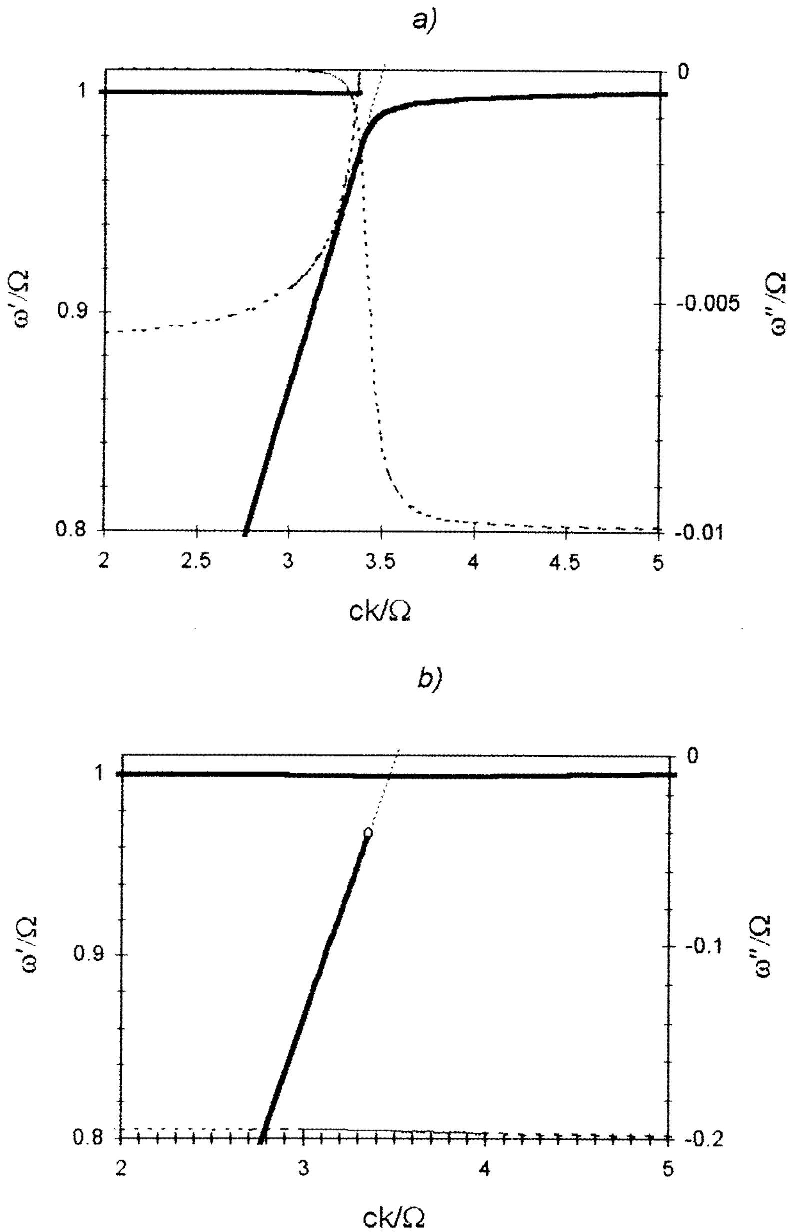

Figure 4. Dynamics of the SP and ASP spectra (ω') (solid line), SP damping (ω'') (dashed line), and ASP damping (ω'') for various values of $\aleph$ and ν/Ω (see the explanation in the text).

In other words, the influence of dissipation on the SP dispersion properties is manifested in new interesting features. When ν exceeds a critical value (see (3.17)), the SP dispersion curve is split into two branches. One of these practically coincides with the 'light line' ($\omega' = ck/\sqrt{\varepsilon}$) and has the end-point of the spectrum $\mathrm{Re}\, p = 0$. At that point the SP field is delocalized. It is easy to see that the threshold condition for the appearance of an ASP

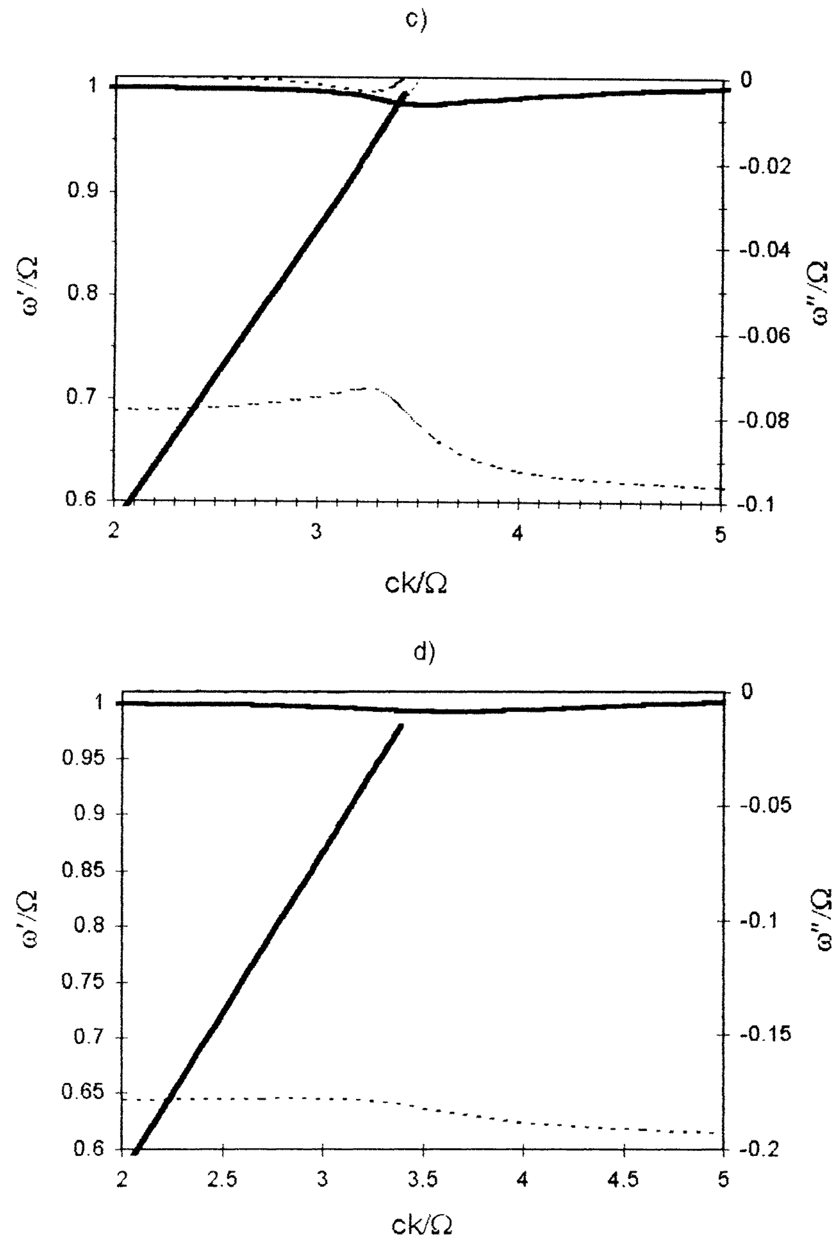

Figure 4. (Continued)

(additional surface polariton) in a high magnetic field is determined by the fine-structure constant and the quantized value of the Landau-level filling factor. Thus, the threshold condition for the existence of ASP in a high magnetic field is quantized due to the Hall quantization. The conductivity spatial dispersion of the 2DES in high magnetic fields (2.14) does not significantly influence either the ASP threshold condition or the ASP spectrum and damping.

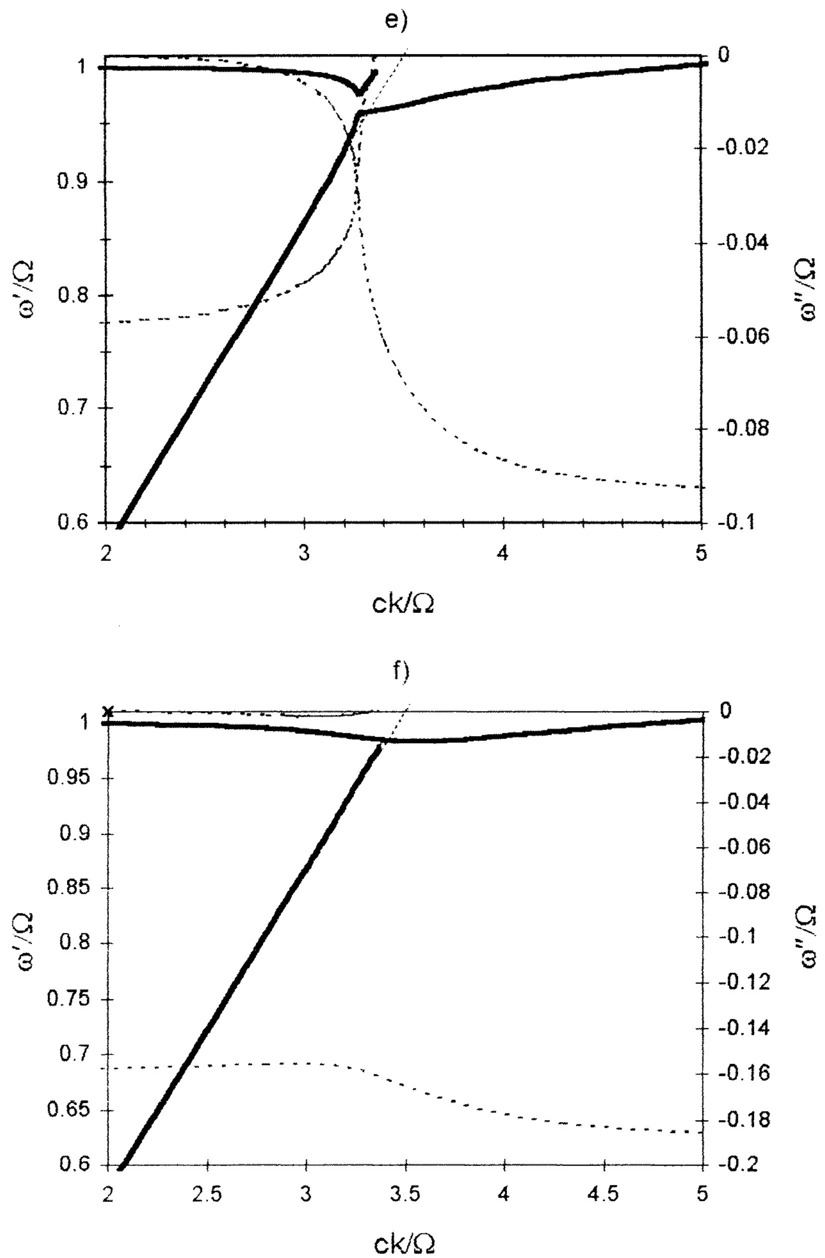

Figure 4. (Continued)

The dispersion curve of the additional (dissipative) SP is shown in figure 4.

Thus, the observation of such a wave specifies the relaxation frequency to an accuracy of up to the fine-structure constant α.

Figure 4(a) ($\aleph = 1$; $\nu/\Omega = 0.01$) shows that at such values of the relaxation frequency ν an ASP appears, which has an end-point of the spectrum defined by the condition Re $p = 0$. The SP damping increases sharply near the CR, where the SP drastically decelerates. The damping of the ASP is sharply decreased and approaches zero, when it tends to the end-point of the spectrum. The ASP exists on the left-hand side of the 'light line' $\omega = kv_d$, being in fact a delocalization wave because it is 'weakly pressed' to the 2DES. The SP is a proper surface wave, which is 'strongly pressed' to the 2DES far from the principal mode (the 'light line' $\omega = kv_d$).

Figure 4(b) ($\aleph = 1$; $\nu/\Omega = 0.2$) shows the spectra of the ASP and SP, when ν is increased. At such values of ν the spectrum is essentially modified. A gap opens in the spectrum. This gap is brought about by the end-point in the spectrum of the low-frequency SP ($\omega < \Omega$) and the blending of the ASP with the decelerated SP. The low-frequency SP becomes a completely delocalized wave; it is practically not connected to the 2DES. The damping of the new blended mode (ASP and SP) has a fixed value ν/Ω over a wide range of the wavenumbers k.

Figure 4(c) ($\aleph = 5$; $\nu/\Omega = 0.1$) shows the spectral curves for lower magnetic fields (when the Landau-level filling factor is $\aleph = 5$). The ASP is blended with the decelerated part of the SP near the principal CR. The low-frequency part of the principal mode of the SP ($\omega = kv_d$) is separated from the ASP, and its spectrum ($\mathrm{Re}\,\omega = \omega' = \omega(k)$) ends at the point where $\mathrm{Re}\,p = 0$. The principal mode $\omega = kv_d$ is 'weakly pressed' to the 2DES for all the values of Ω and k because $\mathrm{Re}\,p \ll \mathrm{Im}\,p$. But the decelerated parts of the SP and ASP (the upper curve in 4(c)) are 'strongly pressed' to the 2DES, since for this part of the spectrum $\mathrm{Re}\,p \gg \mathrm{Im}\,p$. In other words, the slow SP and ASP are proper surface waves.

Figure 4(d) ($\aleph = 5$; $\nu/\Omega = 0.2$) shows a picture of spectral curves when the relaxation frequency ν is greater than that of figure 4(c). The end-point for the principal SP mode ($\omega = kv_d$) is moved down off the ASP and slow SP spectral curve. In this picture only the real parts of the spectral curves cross, while the imaginary parts ω'' of the frequencies assume different values [25].

The spectral picture of the ASP changes crucially at larger values of the Landau-level filling factor (see figure 4(e): $\aleph = 10$; $\nu/\Omega = 0.1$). First, the curves of the ASP and slow SP are separated, and second, the ASP curve acquires an end-point of the spectrum ($\mathrm{Re}\,p = 0$). It is significant that the spectrum shows an anomalous (negative) dispersion near the CR and an end-point of the ASP. At high values of the relaxation frequency (see figure 4(f): $\aleph = 10$; $\nu/\Omega = 0.2$) the picture of the dispersion curves is of the same kind as in figure 4(d), when $\aleph = 5$ and $\nu/\Omega = 0.2$.

The curves for the SP damping in the series of pictures in figure 4 are qualitatively similar. The negative damping of the SP becomes essential and is of order ν/Ω near the CR, where the SP is drastically decelerated. The damping of the ASP is sharply diminished in the vicinity of the point where $\mathrm{Re}\,p = 0$, and tends to zero. The damping of the principal SP mode ($\omega = kv_d$) becomes vanishingly small when the relaxation frequency ν is increased. This is due to the fact that the 2DES has a small conductivity (it transforms to a dielectric) and the surface wave is separated from the 2DES to become a quasibulk mode.

4. Conclusion

In conclusion, we have calculated explicitly the collective-mode spectrum, damping and polarization in a 2DES in a high magnetic field—the slow surface polaritons, where the quantization is essential. We used the Wigner distribution function formalism for the calculation of the spatial and frequency dispersion of conductivity for the 2DES in a high magnetic field. Near the CR the phase velocity of a SP drastically slows down and the group velocity of SP is quantized into fundamental steps whose magnitude is determined by fine-structure constant and the integer Landau-level filling factor. In other words, in a high magnetic field under the quantum Hall effect conditions, the dispersion curves of the SP in a 2DES are also quantized. Besides this, due to the spatial dispersion of the conductivity at high frequencies, the group velocity of the SP becomes negative (anomalous dispersion) near the CR subharmonics. When the relaxation frequency exceeds the critical threshold, condition (3.17), in the 2DES, an ASP arises—a new mode of surface electromagnetic oscillation. As a consequence, the dispersion curves of the SP and ASP change significantly—the spectrum end-points appear and modes become confluent.

To conclude, it should be emphasized that the phase velocity of the SP takes a remarkably small value near the CR. In other words, the 2D electronic layer under the QHE conditions is an effectively decelerating system. This fact can be used for various applications in microelectronics. For example, it can be used for the excitation of surface electromagnetic waves by a beam of charged particles passing near a 2D electronic layer and for efficient conversion of the beam energy into the energy of waves.

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and [3]). But the quantization of high-frequency Hall and longitudinal conductivity exists even at high frequencies. This leads to the line singularities of CR and to the appearance of unusual novel electromagnetic excitation—the slow SP in the 2DES. Therefore in this paper we will use the term

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